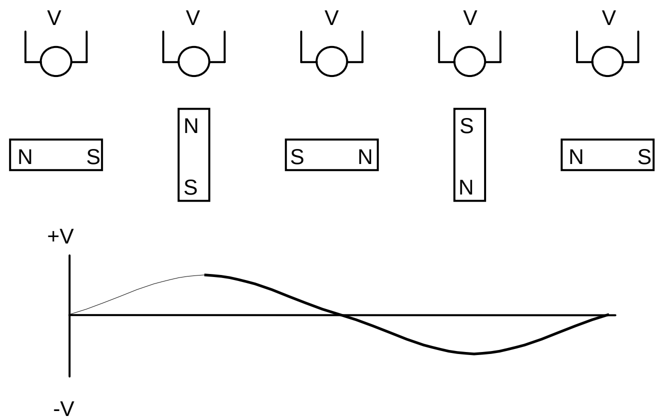
Electric Machinery

Alternating current is created in a coil of wire by a magnet rotating very close to the wire. As the magnetic pole distance varies, the magnitude of voltage induced on the coil changes.

The chart illustrates the magnet at four positions with the fifth position the same as the starting point.



Models

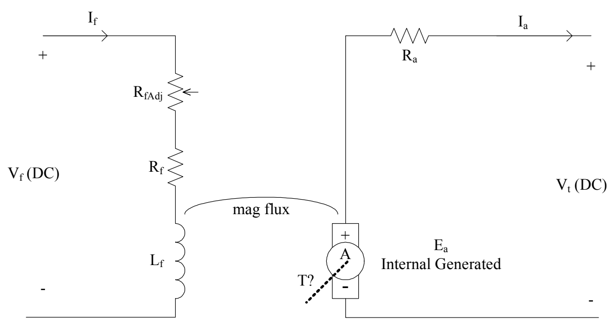
Place model of machine into circuit – two port networks – then perform circuit analysis.

What is the difference between the machines? There are four fundamental classes – DC, synchronous, induction, and transformer.

The input energy and output energy determine the use. A motor has electrical in and mechanical out. A generator has mechanical in and electric out. The same machine can be used in either form. It simply depends on the driver input and the driven output.

- Generator: Mechanical In – Electrical Out
- Motor: Electrical In – Mechanical Out
- Transformer: Electrical In – Electrical Out

DC Machine



Need curve of $I_f - vs - E_A$

$$\lambda = Li = \varphi \mathcal{R}$$

$$V_f = I_f (R_f + R_{fadj})$$

$$E_A = K\varphi\omega$$

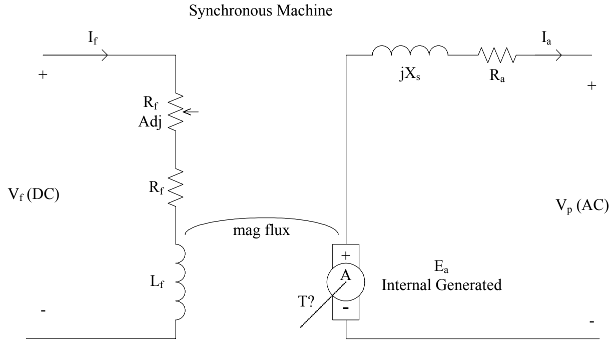
$$\tau = K\varphi I_f$$

$$\frac{E_A}{E_{A0}} = \frac{n}{n_0}$$

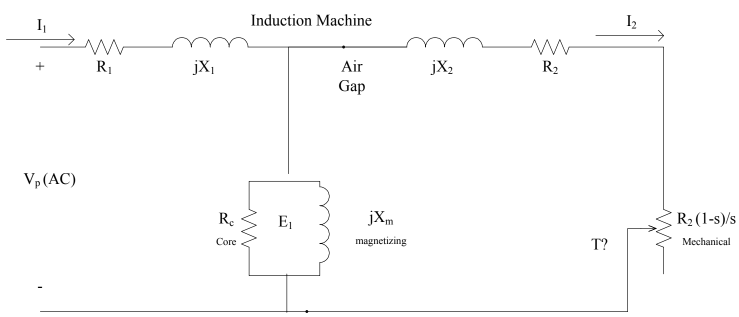
$$V_t = E_A - I_A R_A$$

$$P = E_A I_A = \tau\omega$$

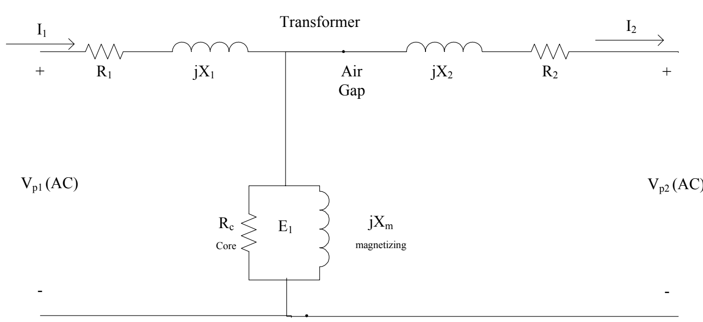
Synchronous Machine



$$V_t = E_A - I_A(R_A + jX_s)$$

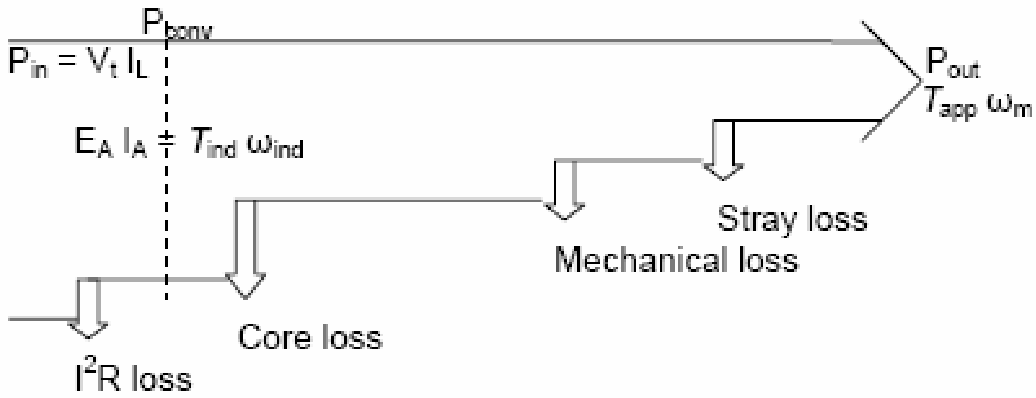


- No load test
  - Set rated voltage, freq
  - Reduced I
  - Read I through core
- Blocked Rotor Test
  - Set rated current, reduced V
  - Read reduced V
  - I through rotor

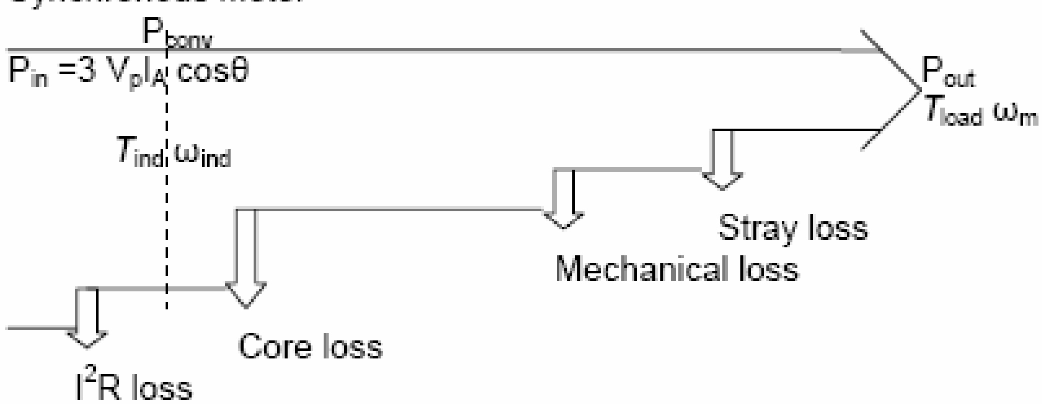


- Open Circuit Test
  - same as no load
- Short Circuit Test (short secondary)
  - Same as blocked

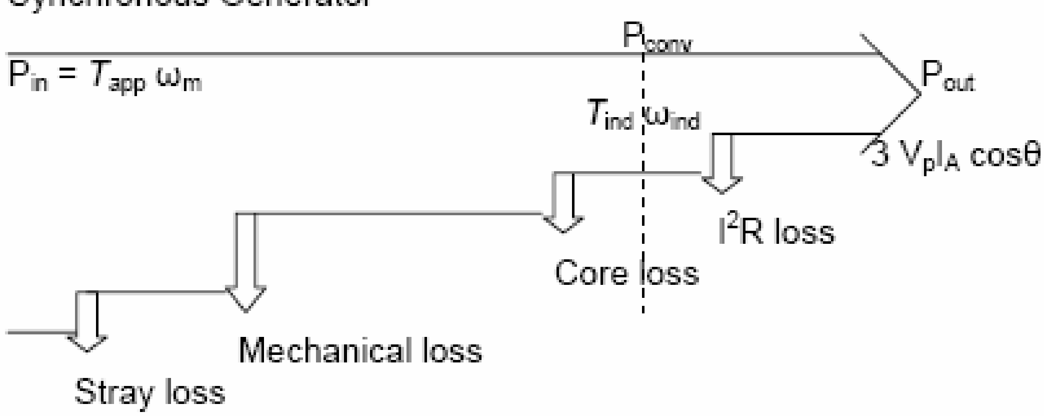
DC Motor

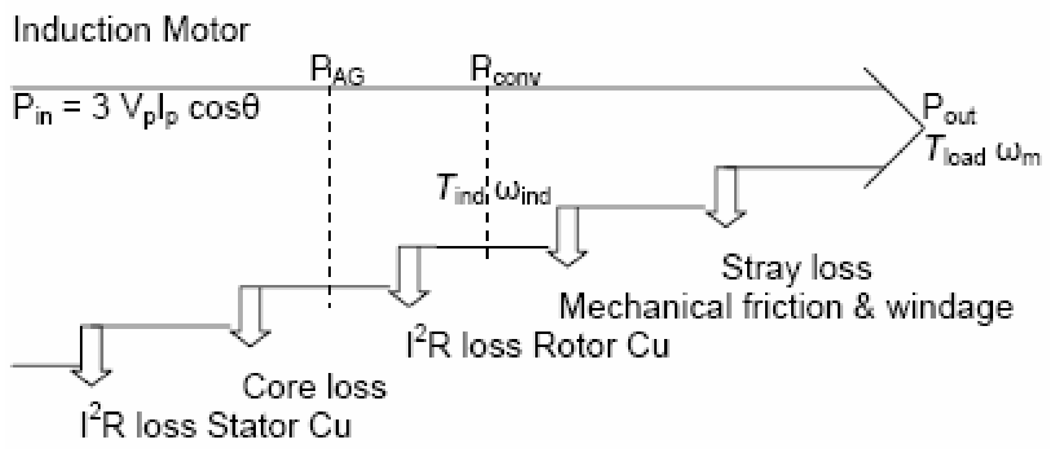


Synchronous Motor



Synchronous Generator





EQUATIONS

- LOSSES
  1. Copper Losses (I^2R)
     - DC Machine - $P_A = I_A{}^2 R_A \qquad P_f = I_f{}^2 R_f$
     - AC Machine - $P_s = 3 I_A{}^2 R_A \qquad \mathrm{P_r} = I_f{}^2 R_f$
  2. Core Losses – Hysteresis & Eddy
     - $\frac{E_m}{R_c}$
  3. Mechanical losses – friction & windage
     - No load rotational = (mechanical losses + core losses)
     - ~Proportional to n^3
  4. Stray losses
     - miscellaneous - ~1% of output power
  5. Brush losses (DC Only)
     - $P_{BD} = V_{BD} I_A \qquad (V_{BD} \approx 2V)$

CONVERSIONS

- Rotate a coil inside a magnetic field develops a voltage
- $E_{\substack{induced \\ armature}} = K\varphi\omega$
  - φ = flux
  - $l = \frac{d\varphi}{dt}$
- $\tau_{induced} = K\varphi I$
- $\omega = 2\pi n$
  - n = speed in rpm
- $E_A = K'\varphi n$
  - $K' = \frac{Z_p}{60a}$ - a constant of machine design

η= efficiency = $\frac{P_{out}}{P_{in}} * 100 = \frac{P_{in} - P_{loss}}{P_{in}} * 100$

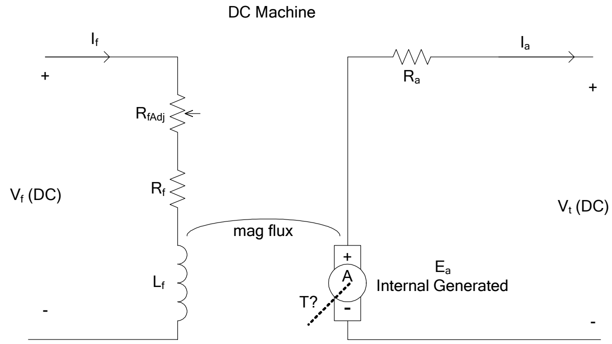
Generator Formulas

- Mechanical Power Converted
  - $P_{conv} = \tau_{shaft}\omega_m$
- Electrical Power Converted
  - $P_{conv} = P_{in} - loss_{stray} - loss_{mechanical} - loss_{core}$
  - $P_{conv} = E_A I_A$
- Output Power
  - $P_{out} = P_{conv} - loss_{I^2R}$
  - AC Machine - $P_{out} = 3V\varphi = \sqrt{3}V_L I_L \cos\theta$
  - DC Machine - $P_{out} = \sqrt{3}V_L I_L$
  - $e_{ind} = N\frac{d\varphi}{dt}$
  - For AC, $e = N\varphi\omega \sin\omega t$
- RMS Voltage for 1φ
  - $E_{\max} = N_c\varphi\omega = \sqrt{2}\pi N_c\varphi f$

- $E_A = \frac{E_{\max}}{\sqrt{2}}$ $E_x = E_A$ $E_y = \sqrt{3}E_A$

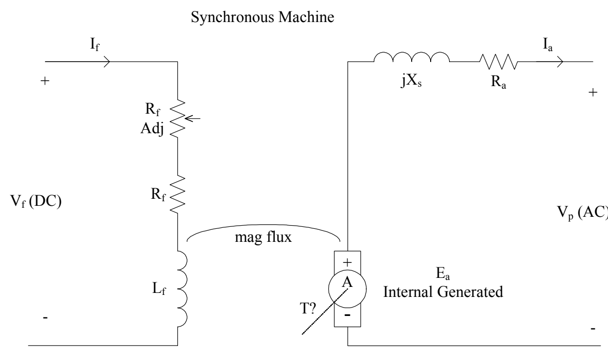
- # poles – p $p/2$ repetitions in one rotation
- Frequency $f = frequency$
  - $f_{elec=} \frac{p}{2} f_{mech}$
  - $f_m = \frac{n_m}{60}$
  - $f_e = \frac{n_m}{60}\frac{p}{2} = \frac{n_{mp}}{120}$
- Voltage Regulation for generators and sources
  - $V_R = \frac{V_{n1-}V_{f1}}{V_{f1}}100\%$
- Speed Regulation for motors
  - $S_R = \frac{n_{n1} - n_{f1}}{n_{f1}} \times 100\%$
  - Positive S_R means speed drops with load

AC Machine



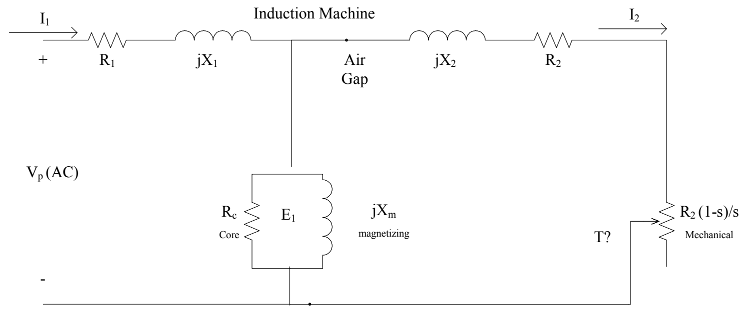
- Synchronous Machine
  - Field by separate DC source (rotor)
- Induction
  - Field by magnetic induction (rotor)
- Armature on stator - 3φ currents relate to magnetic field in rotor
- 120º equal magnetic current
- 2 pole (1N-1S)

Synchronous Machine – model is per phase

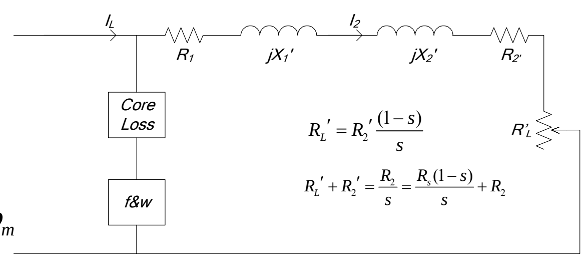


- $E_A = V_\varphi - jX_S I_A - R_A I_A$
- $I_A \cos\theta = \dfrac{E_A \sin\delta}{X_S}$
- $P_{out} = \dfrac{3V_\varphi E_A Sin\delta}{X_S}$
  - X_s=Synchronous Reactance
  - δ = torque angle: max τ @ 90º
  - $\delta \approx 15 - 20^\circ$
- $P = \tau\omega$ $\qquad \tau = \dfrac{3V\varphi E_A \sin\delta}{\omega_m X_s}$
- $\tau = K\varphi I_f$
- $E_A = K\varphi\omega = K_1 i_f \omega$
- $V_f = I_f (R_f + R_{fadj})$
- $V_t = E_A - I_A(R_A + jX_s)$
- $\lambda = Li = \varphi\mathcal{R}$

Induction Machine



- No DC Field
- Rotor field has short bars & induced
- Slip
  - Slip Speed – $n_{slip} = n_{sync} - n_m$
  - Slip (per unit – $s = \dfrac{n_{slip}}{n_{sync}} \times 100 = \dfrac{n_{sync} - n_m}{n_{synce}} \times 100$
    - $s = \dfrac{\omega_{sync} - \omega_m}{\omega_{sync}} \times 100$
    - $s = 0 \Rightarrow$ rotor @ sync speed
    - $s = 1 \Rightarrow$ rotor stationary (locked)
  - $n_m = (1 - s) n_{sync}$
- Losses – power & torque – motor



  - $P_{in} = \sqrt{3} V_T I_L \cos\theta$
  - $P_{air\,gap} = P_{in} - P_{stator\,cu} - P_{core}$
  - $P_{conv} = P_{air\,gap} - P_{rotor\,cu} = \tau_{ind} \omega_m$
  - $P_{out} = P_{conv} - P_{f\&w} - P_{stray} = \tau_{load} \omega_m$
- Power & Torque
  - $P_{Stator\,cu} = 3 I^2 R_1$
  - $P_{core} = 3 \dfrac{E_1^{\,2}}{R_c}$
  - $P_{air\,gap} = P_{in} - P_{s\,cu} - P_{core}$
    - Only real element to consume
  - $P_{ag} = 3 I_2^{\,2} \dfrac{R_2}{S}$
  - $P_{rotor\,cu} = 3 I_2^{\,2} R_2 = s P_{ag}$
- Developed mechanical power

$$P_{conv} = P_{ag} - P_{r\,cu}$$

- $$= 3I_2{}^2 \frac{R_2}{s} - 3I_2{}^2 R^2$$

  $$= 3I_2{}^2 R_2 \frac{(1-s)}{s}$$

- $$P_{out} = P_{conv} - P_{f\&w} - P_{misc}$$

Per Unit Notation

- Per unit notation is used to reduce the complexity when working with circuits that have multiple voltage levels.
- Both Ohm's law and the power relationship permit a third term to be calculated from only two terms.
- Two parameters are selected as the reference or base values. These are generally S and V. A different base V is used on each side of a transformer.
- The base current and base impedance can be determined from these two values
    - $I_{base} = \frac{S_{base}}{V_{base}}$
    - $Z_{base} = \frac{V_{base}^{2}}{S_{base}}$
- All the circuit equipment voltages and currents are then converted to per unit (percentage) values beore normal circuit calculations are made
    - $S_{pu} = \frac{S_{equip} * 100}{S_{base}}$
    - $V_{pu} = \frac{V_{equip} * 100}{V_{base}}$
    - $I_{pu} = \frac{I_{equip} * 100}{I_{base}}$
    - $Z_{pu} = \frac{Z_{equip} * 100}{Z_{base}}$
- As an example, transformer impedance is usually rated in per unit values. To find the actual impedance, combine the above equations
    - $Z_{equip} = \left(\frac{Z_{pu}}{100} \right) Z_{base}$
    - $Z_{equip} = \left(\frac{Z_{pu}}{100} \right) \frac{V_{base}^{2}}{S_{base}}$
- An example illustrates the relationship between per unit values and short circuit capability
    - Transformer, S_{base}=10kVA, V_{base}=120, Z_{pu}=2%

- $Z_{equip} = \frac{\left(\frac{2}{100}\right)120^2}{10000} = 0.0288\Omega$
- $SCC = 10000\left(\frac{100}{2}\right) = 5000kVA$
- $I_{sc} = \frac{V}{Z_{equip}} = \frac{SCC}{V_{base}} = \frac{V_{base}}{Z_{equip}} = 4167A$

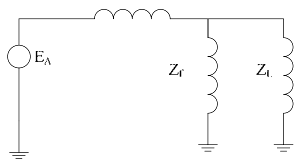
Short Circuit Considerations

- A short circuit condition differs from normal current operations only by virtue of an accidental decrease in the circuit impedance. The decrease in impedance is caused by a fault.
- The power source is generally rated by a short circuit capacity (SCC) rating in volt-amps. This is the product of the pre-fault voltage and the post-fault current. Short circuit current is restricted only by the source impedance, since the load greatly reduced.
  - $V_{A_{SCC_1}} = V_{pre} I_{sc}$
  - $V_{A_{SCC_3}} = \sqrt{3} V_{pre} I_{sc}$
- With the short circuit capability and the voltage rating, the source impedance can be determined. The impedance calculated is for each phase, if the system is three-phase.
  - $Z_{source} = \frac{V_p^{\ 2}}{SCC}$
- The SCC of a magnetic device, such as a transformer or machine, can be found using the percent impedance (Z_{pu}) and the device rating
  - $SCC = kVA\left(\frac{100}{Z_{pu}}\right)$
- The available fault current is restricted by the source fault current and the transformer turns ratio.
  - $N = \frac{V_{primary}}{V_{secondary}}$
  - $I_{SC_{secondary}} = I_{SC_{primary}} N$
- The available fault current is also restricted by the SCC of the transformer
  - $I_{SC_3} = \frac{SCC_3}{\sqrt{3} V_{line}}$
  - $I_{SC_1} = \frac{SCC_1}{V_{line}}$
- The available fault current is the smaller value that is calculated using the two methods above. Other impedance in the wiring will further restrict the fault current.

- $I_{SC} = \dfrac{V_{pre}}{Z_{fault}}$

- Short circuit contribution from induction machines continues after a fault. Inertia causes the machine to continue turning with a collapsing magnetic field. This results in approx 25% of the machine's capability contributing to the fault current.

Fault Analysis

- Commonly called short-circuit study
- Fault current differs form normal current only by an accidental decrease in circuit Z.
- 

- Under fault conditions, the load (Z_L) may be 1 or more Ohm, while the fault is ~0.0001 ohm.
- The resulting Z - $\frac{1}{Z_T} = \frac{1}{0.0001} + \frac{1}{1} = 1 \times 10^4 + 1$
- The load is negligible.
- $I_{fault} = \frac{V_A}{Z_T} \approx \frac{V_A}{Z_f}$
- Realistically, the Z_s will provide a significant restriction on fault current

1. Need complete one-line diagram
2. Convert to per unit (percent)
3. Normally pick S_b & V_b, and then calculate I_b & S_b
   - $I_b = \frac{S_b}{V_b}$
   - $Z_b = \frac{V_b}{I_b} = \frac{|V_b|^2}{S_b} = \frac{\left(\sqrt{3}V_b\right)^2}{3S_b}$

   a

- Can use either per phase values or line values for 3 phase calculation
- Do all calculations on single phase basis for Z

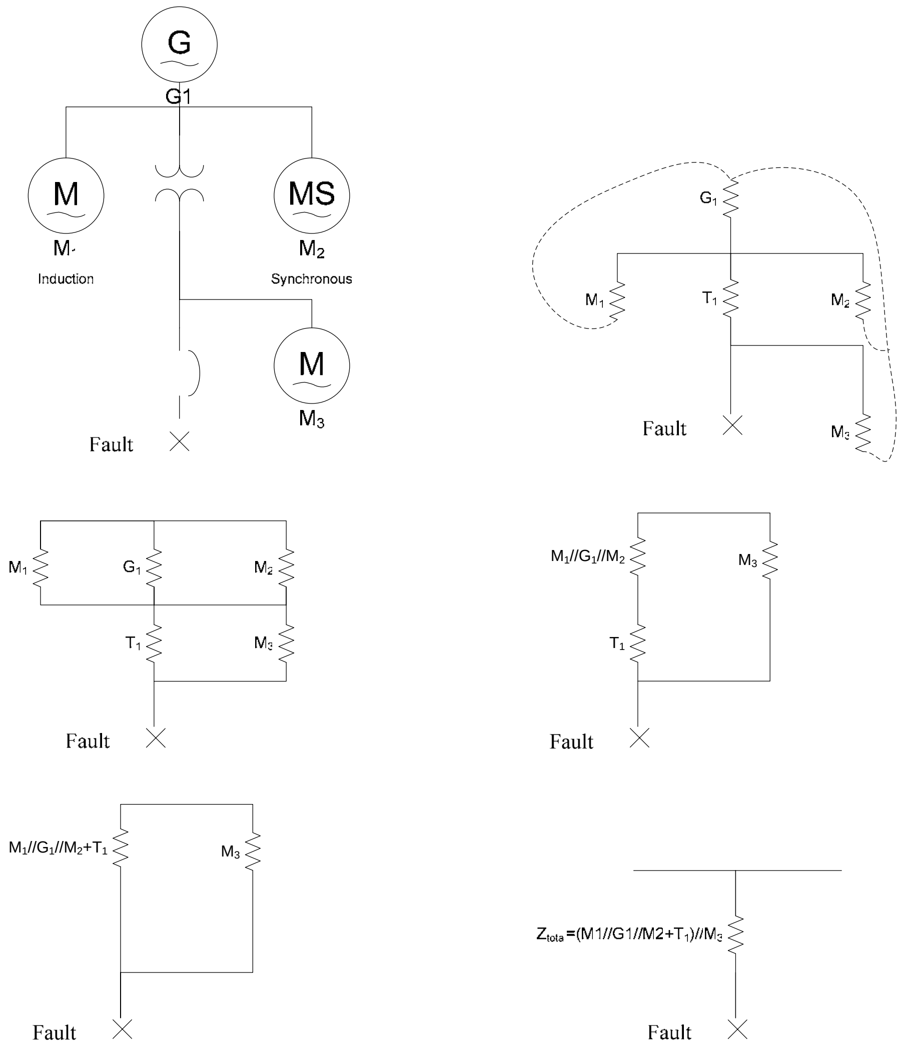
Short Circuit Study (Symmetrical Components)

1. Draw single line diagram w/ all sources of fault current, such as generators & motors, and utility connections.
2. Replace all components, including reactance, with resistors (impedance) symbol, and label with letters.
3. Show all transformer secondary feeding induction motors, whether motors are indicated or not.
4. Join all components by "infinite bus" (neutral)
5. The source is not the infinite bus, but is simply another reactance.
6. Rearrange impedances into series & parallel.
7. Reduce to single Z.
8. Convert Δ blocks to star to further reduce (Thevenin Z)

Ratings & Reactances

1. Momentary – use all induction motors and subtransient (X_d'') reactances
2. Interrupting – neglect branches w/ pure induction motors and use only transient (X_d') reactances, except below 60?
3. Assymetrical – use multiplier from tables
4. Rules of Thumb
   - $X_d \approx 1.0$
   - $X_d' \approx 0.33$
   - $X_d'' \approx 0.25\ for < 600V$
   - $X_d'' \approx 0.2\ for > 600V$

- Example



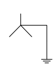
Calculate Thevenin Z

Use pre-fault V @ fault to find I

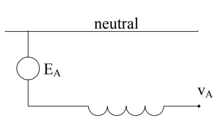
Symmetrical Components

- Convert 3φ to X&Y axis
- $\alpha = -0.5 + j0.866$
- $\alpha^2 = -0.5 - j0.866$
- Sequence Currents
  - $I_+ = \frac{1}{3}(I_A + \alpha I_B + \alpha^2 I_C)$
  - $I_- = \frac{1}{3}(I_A + \alpha^2 I_B + \alpha I_C)$
  - $I_0 = \frac{1}{3}(I_A + I_B + I_C)$
- Phase Currents
  - $I_A = I_{A+} + I_{A-} + I_{A0}$
  - $I_B = I_{B+} + I_{B-} + I_{B0}$
  - $I_C = I_{C+} + I_{C-} + I_{C0}$
- Symmetrical components are used to take any unbalance combination of V & I and make them operate as balanced 3φ model.
- Only use is to aid the algebra. Symmetrical components are not “real”.

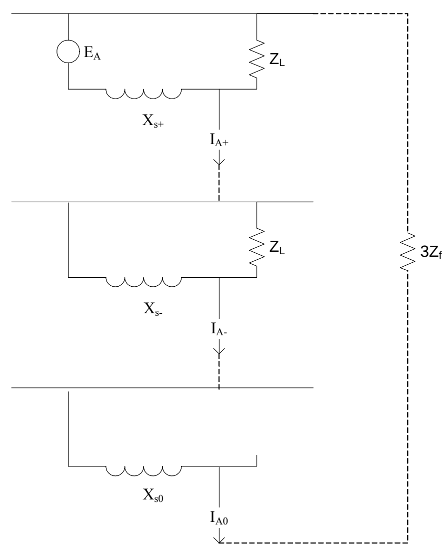
Unbalanced Faults

- Previous development was for a 3 phase fault
    - $I_A + I_B + I_C = 0$
- Unbalanced conditions are redefined in terms of 3 components.
    - Positive sequence (+, p, 1)
        - System with sources rotating
        - “Normal” conditions
    - Negative sequence (-, n, 2)
        - Same as positive without sources
        - Z may have different value
    - Zero Sequence (0, Z, 0)
        - Ground Path
        - Δ No ground
        -  No Ground
        -  Ground path
- Draw Three circuits
    - Leave sources in positive
    - Make negative w/o sources
    - Zero indicates ground paths
- Connect with fault impedances for each compenent
    - $3Z_f$ for pos, neg & zero

Rotating Machine Model

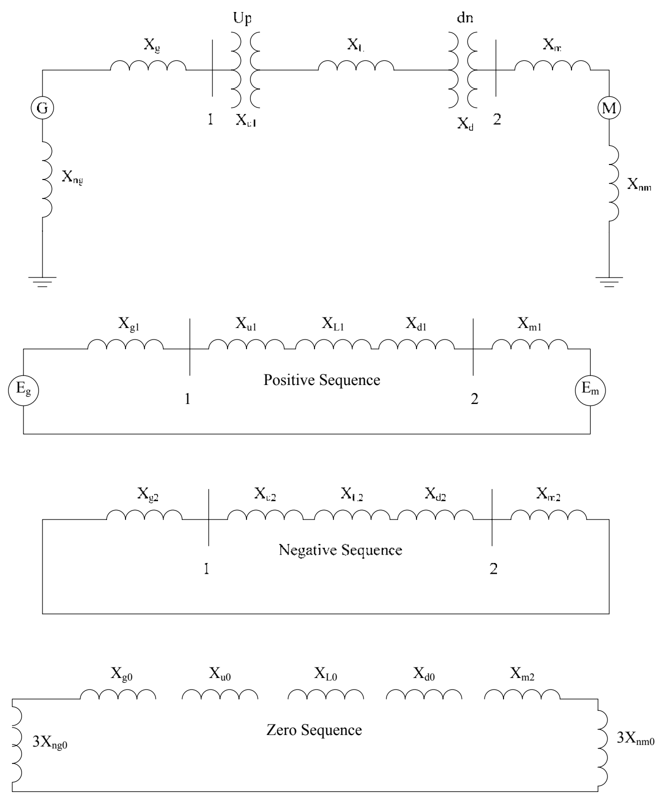


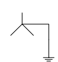
- $V_+ = E_A - Z_+ I_+$
- $V_- = 0 - Z_- I_-$
- $V_0 = 0 - Z_0 I_0$



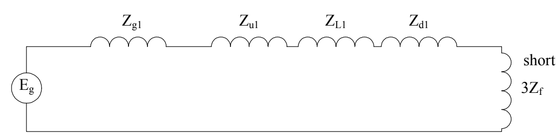
- For 3φ, use positive sequence only
    - $I_A + I_B + I_C = 0$
    - $I_A = I_{A+} + I_{A-} + I_{A0}$
- For 1φ
    - $I_{A+} = \frac{1}{3} I_A$, since $I_{A+} = I_{A-} = I_{A0}$
- The current is drawn at fault
- Z_0 will be different since transformer & machine ground path may not be connected
- Z_0 motor or gen = $3Z_n$
    - Z_0= 0 for connected neutrals
    - $3Z_n$=3 times impedance of any phase
    - Use in place of source voltage
- Assumption
    - $Z''_{d0motor} = \frac{1}{2} Z''_{d0}$

Example

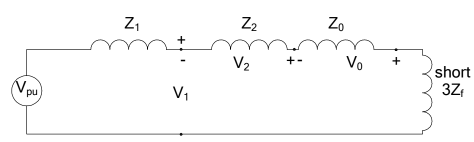


Connection on transformer to ground is  , else leave open

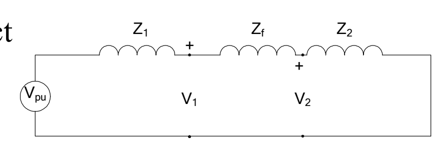
1. Three phase fault @ 2 – use positive sequence only
   - $V_0 = V_2 = 0$
   - $I_1 = \dfrac{V_f}{Z_{eqt}}$
   - $I_0 = I_2 = 0$



2. Single phase line to ground @2 – use pos, neg, zero sequence in series
   - $V_0 + V_1 + V_2 = 3Z_f I_1$
   - $I_1 = \dfrac{V_{fault}}{Z_1 + Z_2 + Z_0 + 3Z_f}$
   - $I_1 = I_2 = I_o$



3. Line to Line
   - Use pos in parallel w/ neg, connect with Z_f



- $I_1 = \dfrac{V_f}{Z_1 + Z_f + Z_2}$
- $I_1 = -I_2$

4. Double line to ground

- $I_1 = \dfrac{Vf}{(Z_1 + Z_2 \| 3Z_f + Z_0)}$
- $I_1 = I_2 + I_0$