Chapter 9 - Induction

9.1 Introduction

The induction machine follows the Thevenin equivalent for the armature with a magnetizing inductance for the field all within a two-port network. However, the magnetizing circuit cannot be separated, but is an integral part of the machine. The field is on the stator and the armature is on the rotor.

The induction machine is simply a transformer that has the secondary shorted and allowed to spin. As such it is a purely AC machine with no DC field. The rotor field has shorted bars with voltage induced from the stator field.

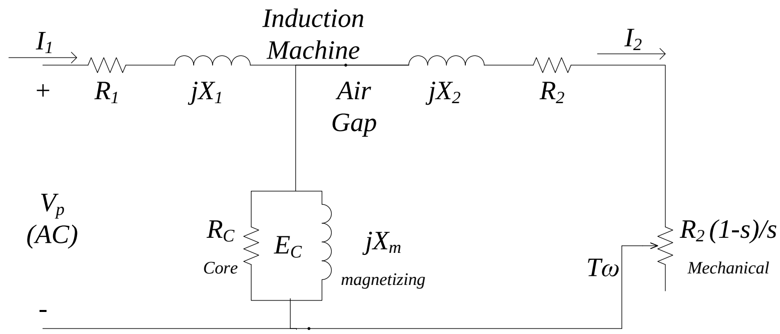
Because of the rotation, additional calculations are required compared to the transformer. The rotor speed varies with load. The difference in the synchronous speed and the mechanical shaft speed is called the slip.

9.2 Model

The model of an induction machine consists of the rotor electrical, stator electrical, core magnetic, and mechanical circuits.

9.2.1 Slip

Slip is the difference in the synchronous speed and mechanical speed divided by the synchronous speed. Slip, *s*, is normalized or per unit.



$$s = \frac{\omega_{sync} - \omega_m}{\omega_{sync}} \times 100$$

$$s = \frac{n_{slip}}{n_{sync}} \times 100 = \frac{n_{sync} - n_{mech}}{n_{synce}} \times 100$$

$$s = 0 \Rightarrow \text{rotor @ sync speed}$$

$$s = 1 \Rightarrow \text{rotor stationary (locked)}$$

$$n_{slip} = n_{sync} - n_m \text{ - Slip Speed}$$

$$n_m = (1 - s) n_{sync}$$

Some models combine the two components of R_2 into one value of R_2/s. Mathematically that is correct, but in application the winding is separated from the mechanical component.

Example slip:

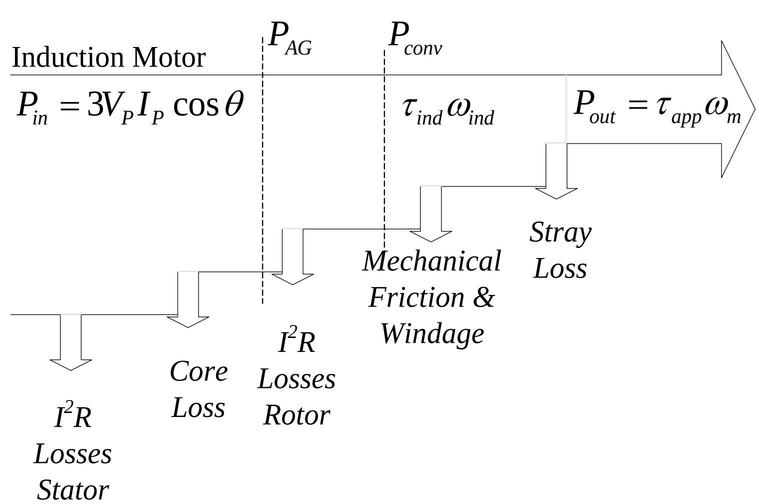
Speed = 1720, s = 0.04 = 4%

Find speed for slip = 5%

9.2.2 Losses

The standard losses are copper, iron, and mechanical. The electrical and magnetic losses are on the stator and the rotor.

The air gap (ag) is the separation of the stator and rotor. The converted power is from electrical to mechanical.



$$P_{in} = \sqrt{3} V_T I_L \cos\theta$$

$$P_{air\,gap} = P_{in} - P_{stator\,cu} - P_{core}$$

$$P_{conv} = P_{air\,gap} - P_{rotor\,cu} = \tau_{ind}\omega_m$$

$$P_{out} = P_{conv} - P_{f\&w} - P_{stray} = \tau_{load}\omega_m$$

9.2.3 Power & Torque

Only real elements consume power and convert it to mechanical energy or heat. This depends on the square of the electrical current and the magnetizing voltage.

$$P_{stator\,cu} = 3I^2 R_1$$

$$P_{core} = 3\frac{E_1^{\,2}}{R_c^{\,2}} R_c$$

$$P_{air\,gap} = P_{in} - P_{s\,cu} - P_{core}$$

$$P_{ag} = 3I_2^{\,2}\frac{R_2}{s}$$

$$P_{rotor\,cu} = 3I_2^{\,2}R_2 = sP_{ag}$$

9.2.4 Mechanical power

The developed mechanical power is dependent on the slip.

$$P_{conv} = P_{ag} - P_{r\,cu}$$

$$= 3I_2^{\,2}\frac{R_2}{s} - 3I_2^{\,2}R^2$$

$$= 3I_2^{\,2}R_2\frac{(1-s)}{s}$$

$$P_{out} = P_{conv} - P_{f\,\&w} - P_{misc(stray)}$$

9.2 Example

Data
Induction motor: 460 V, wye connected, 20 hp, Design A, 27 Amp, 2% slip, X1=X2
Power factor at rating = 0.8, Rotational losses = 10% of converted power

Find the following.

synchronous speed ________________

$$n_{sync} = 120\tfrac{f_e}{p} = 120\frac{60}{6} = 1200$$

shaft speed ________________

$$n_m = (1-s)n_{sync} = (1-0.02)1200 = 1176$$

stator current ______________________

$$I_s = I_L = 27A$$

power input __________________________

$$P_{in} = \sqrt{3}V_T I_L\cos\theta = \sqrt{3}(460)(27)(0.8) = 17.21kW$$

stator copper losses ________________________

$$P_{SCL} = 3I^2R_1 = 3(27^2)(0.5) = 1094W$$

power in air gap ____________________________

$$P_{AG} = P_{IN} - P_{SCL} = 17.21 - 1.09 = 16.12kW$$

rotor current _________________________________

$$P_{AG} = 3I_2^2 \frac{R_2}{s}$$

$$I_2 = \sqrt{\frac{sP_{AG}}{3R_2}} = \sqrt{\frac{0.02(16.12)}{3(0.25)}} = 20.7A$$

power converted ________________

$$P_{Conv} = (1-s)P_{AG} = (1-.02)16.12 = 15.8kW$$

power output (kw) ___________________________

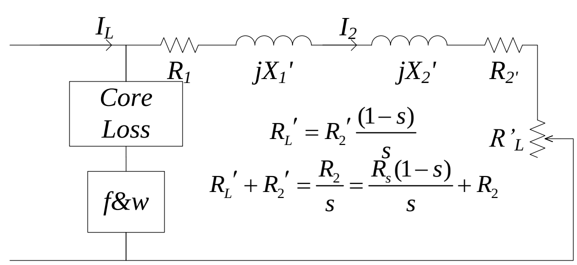
$$P_{OUT} = P_{Conv} - P_{AG} = P_{Conv}(1-0.1) = (.9)15.8 = 14.22kW$$

torque output at shaft speed _________________

$$\omega_m = \frac{1176rev}{\min}\frac{2\pi Rad}{rev}\frac{\min}{60\sec} = 123.15rad/\sec$$

$$\tau_L = \frac{P_{out}}{\omega_m} = \frac{14.22}{123.15} = 115.3Nt-m$$

9.3 Simplified model



Like the transformer, the core losses and mechanical losses can be considered in parallel with the electrical losses. The model is more feasible to calculate.

These relationships allow calculating locked rotor and no load circuit elements of an induction machine.

The results give a combined X for the rotor and stator. These are separated based on the machine design, whether A, B, C, D.

In general, the losses are shared 40/60 to 50/50 between the rotor and stator.

9.3 Example

Data

Induction motor: 460 V, wye connected, 20 hp, Design A, 27 Amp, 2% slip, X1=X2

No load test: 460 V, 4.4 A, 480 W, 60 Hz
Locked rotor: 46 V, 13 A, 880 W, 10 Hz
Dc test: 11.5 V, 27 A

Find the following:

phase voltage ________________________

$$V_P = V_T/\sqrt{3} = 460/\sqrt{3} = 266V$$

locked rotor Z magnitude at test frequency ________________________________

$$Z_{LR} = \frac{V_P}{I_P} = \frac{V_T/\sqrt{3}}{I_P} = \frac{46/\sqrt{3}}{13} = 2.04\Omega$$

locked rotor angle between voltage and current _______________________________

$$\cos\theta = \frac{P}{S}$$

$$\theta = \cos^{-1}\frac{P_{IN}}{\sqrt{3}V_T I_L} = \cos^{-1}\frac{880}{\sqrt{3}(46)(13)} = \cos^{-1}(.85) = 31.8^0$$

locked rotor resistance ______________________________________

$$\cos\theta = \frac{R}{Z}$$

$$R = Z\cos\theta = 2.04(.85) = 1.732$$

locked rotor reactance at test frequency _______________________________

$$\sin\theta = \frac{X}{Z}$$

$$X = Z\sin\theta = 2.04(31.8) = 1.07$$

locked rotor reactance at rated frequency _________________________________

$$X = 2\pi fL$$

$$L = \frac{X_{test}}{2\pi f_{test}} = \frac{X_{rate}}{2\pi f_{rate}}$$

$$X_{rate} = \frac{2\pi f_{rate} X_{test}}{2\pi f_{test}} = \frac{f_{rate}}{f_{test}} X_{test} = \frac{60}{10}(1.07) = 6.45$$

stator resistance _________________________

$$R_{measured} = \frac{V_P}{I_P} = \frac{11.5}{27} = 0.426$$

$$R_{measured} = R_{Wye} = 2R_1$$

$$R_1 = \frac{R_{Wye}}{2} = \frac{0.426}{2} = 0.213\Omega$$

rotor resistance ________________

$$R_{LR} = R_1 + R_2$$

$$R_2 = R_{LR} - R_1 = 1.732 - 0.213 = 1.52\Omega$$

stator reactance ______________________________

$$X_{LR} = X_1 + X_2$$
$$X_1 = X_2 = 0.5X_{LR} = 0.5(6.45) = 3.22$$

rotor reactance ______________________________

Same as stator reactance

no load reactance ____________________________

$$X_{NL} = \frac{V_P}{I_P} = \frac{V_T/\sqrt{3}}{I_P} = \frac{460/\sqrt{3}}{4.4} = 60.36\Omega$$

magnetizing reactance _______________________

$$X_{NL} = X_1 + X_M$$
$$X_M = X_{NL} - X_1 = 60.36 - 3.22 = 57.14$$

9.4 50 / 60 Hz Operations

9.4.1 International

As international trade proliferates, there is increasing use of equipment on both 50 and 60 Hz. Sixty Hz is primarily used in the Western Hemisphere as well as Philippines, Korea, Taiwan, Hong Kong, Guam, Okinawa, Liberia, and in some areas of Japan and Saudi Arabia. Europe and the remainder of Asia and Africa typically operate at 50 Hz. Aviation equipment is rated at 400 Hz. The primary benefit of higher frequency is smaller physical size. As the frequency changes, the performance also changes.

With few exceptions, the single-phase voltage of 60 Hz countries systems is a nominal 120 Volts, while the 50 Hz systems are nominally 220 to 240 Volts. The international single-phase voltages are one phase of a three-phase grounded wye system. That simply means that the 3-phase voltage is 1.732 times the single phase.

$V(3\varphi) = 1.732\ V(1\varphi)$

System voltage is the nominal rating at the secondary of the transformer. Typically 5% voltage drop is assumed in the wiring. Therefore, motor voltages are 5% less than the system. The ratings are often rounded to the nearest whole number. For example, on a 60 Hz system, the transformer voltage is 480 V and the motor voltage is rounded to 460 V.

The corresponding voltages for 60 and 50 Hz are given in the table.

60 Hz 3-ph V	50 Hz 3-ph V	50 Hz 1-ph V
498	415	240
480	400	230
456	380	220

9.4.2 Performance

When using a machine from one system to another, the performance will change. To understand the relationships, it is necessary to review the magnetic, electrical, and mechanical conversion characteristics of machines.

Electrical voltage (V) equals the machine constant (K) multiplied by the magnetic flux (φ) and the angular speed (ω).

$V = K\ \varphi\ \omega$

Mechanical torque (T) equals the machine constant multiplied by the magnetic flux and the current (I).

$T = K \varphi I$

Mechanical power (P) is the product of torque and angular speed.

$P = T \omega$

Angular speed in radians/sec is the circular or cyclic frequency. It is related to 2π times electrical frequency (f) in Hertz. The electrical frequency is the number of magnetic poles times the RPM (revolutions/minute) divided by a constant 120.

$\omega = 2\pi f$

f = RPM * poles / 120

Electrical power is the product of voltage, current, phase factor, and power factor (pf). The phase factor is 1.732 for 3-phase and 1 for 1-phase. The power factor depends on the inductance. It is typically 0.8 for machines.

P = 1.732 V I pf

The key common ingredient to performance is the magnetic flux. First consider the effect of frequency on voltage. Keep the flux constant. Then the voltage to frequency ratio is constant. Hence, voltage is proportional to the frequency.

$V = K \varphi \omega$

$K \varphi = V1 / f1 = V2 / f2$

480 / 60 = 400 / 50

Next consider the effect of frequency change on the current. If the power load is fixed, then the voltage and current product at one frequency equals the product at another. Hence, current is inversely proportional to the new frequency.

P = 1.732 V1 I1 pf = 1.732 V2 I2 pf

V1 I1 = V2 I2

480 * 10 = 400 * 12

Speed is obviously proportional to the frequency, since the number of poles and 120 are fixed.

f = RPM * poles / 120

poles / 120 = f1 / RPM1 = f2 /RPM2

60 / 1800 = 50 / 1500

There is a significant benefit to running at higher speed. The power is proportional to the speed. If the voltage is adjusted for the frequency and the current is allowed to rise to the rated current, then the available power increases proportional to the frequency.

$P = T 2\pi f$

$2\pi T = P1 / f1 = P2 / f2$

1.2 / 60 = 1 / 50

9.4.3 Considerations

1. Running a 50 Hz machine on 60 Hz will cause the flux to increase above its rating. Over saturation, will then cause overheating and burnout.

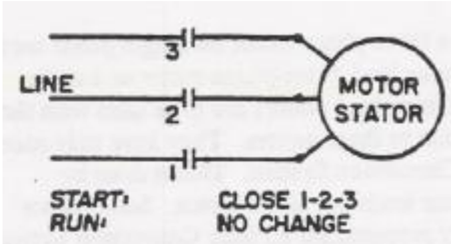
2. Running a 60 Hz machine on 50 Hz will often work, but perhaps not as effectively.

3. If the load is fixed, and the machine is rated 50 Hz, and the 60 Hz voltage is proportional to the frequency, then the current will decrease. The machine is then operating with a service factor increase proportional to the frequency ratio (60 /50 = 1.2).

4. Alternately, if the current is allowed to rise to rating, then the available power from the machine is increased by the frequency ratio (60 /50 = 1.2).

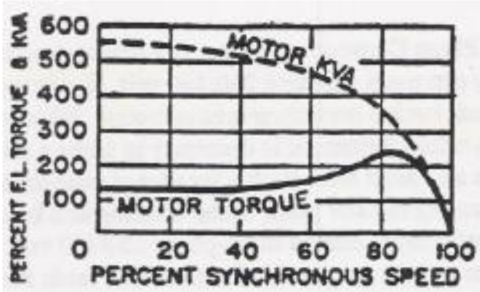
9.5 Induction motor starting

9.5.1 Full Voltage



The method of starting has an important effect on the voltage drop in the system. Several starting methods are available.
The most common method consists of connecting the motor terminals directly across the line. Due to its simplicity and low initial cost, across the line , full voltage starting is used whenever system capacity and mechanical shock considerations permit.

9.5.2 Reduced Voltage - Mechanical



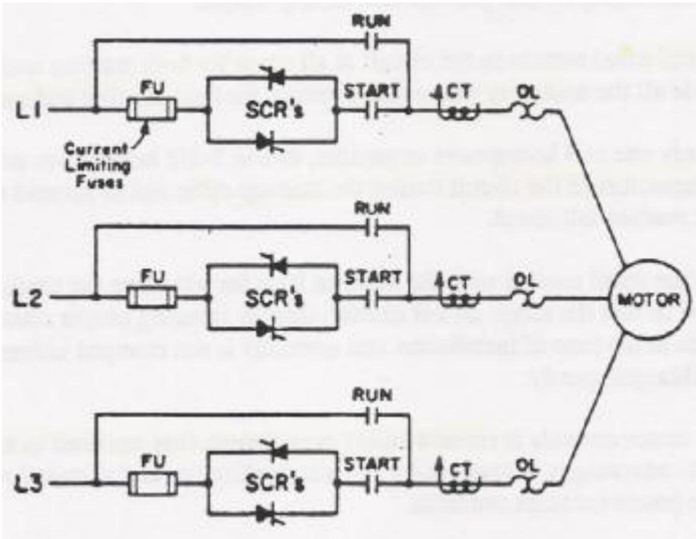
These systems consist of (1) using only part of the motor winding, (2) changing the motor winding connection between delta and wye, or (3) placing an impedance in series with the motor terminals during starting. The impedance may be a resistor or reactor.

Each of the options restricts the current flow to the motor and thereby minimizes the voltage drop on the system. However, it also reduces the starting torque of the motor.

$$T = K \varphi I$$

9.5.3 Reduced Voltage - Solid State

Technology has made this the preferred method. Silicon controlled rectifiers (SCR) control the motor voltage during acceleration. The method offers smooth stepless acceleration as well as selectability of the acceleration current as a percent of full load motor current. Two modes of operation are constant current acceleration and linear timed acceleration.



With constant current acceleration, the starter is designed to maintain the motor current at a constant level throughout the acceleration period. An adjustment is provided to preset this starting current from 200% to 500% of the full load motor current. The time allowed for acceleration is inversely proportional to the square of the current setting. If the motor does not accelerate within this time, the SCR's are turned off and the start contactor is automatically de-energized

The starter with linear timed acceleration uses a closed loop feedback system to maintain the motor acceleration at a constant rate. The required feedback signal is provided by a tachometer coupled to the motor. This starter has three adjustments. *Acceleration time* varies the acceleration ramp reference period from 5 to 30 seconds. *Current limit* sets the maximum level of current from 200% to 500% of the motor full load current. *Breakaway torque* varies the available starting current (torque) up to the current limit setting..

If the actual motor speed during acceleration lags the desired motor speed by more than 15% of the full load speed for a period of 5 seconds, this indicates that the load is too great for the available torque. Then the SCR's are turned off and the start contactor is de-energized.

When the starter is energized, the start contactor closes, and motor acceleration is controlled by phasing-on the SCR's. When the motor reaches full speed the run contactor closes, connecting the motor directly across the line. At this point the SCR's are turned off and the start contactor opens. Under full speed running conditions, the SCR's are turned off and the start contactor opens. The SCR's are out of the circuit. This eliminates SCR power dissipation during the run cycle and guards against possible effects of over voltage transients.

9.6 Exemplars

An exemplar is typical or representative of a system. These examples are representative of real world situations.

Practice Problem 1

SITUATION:

A refinery is planning to install a compressor-expander connected to an induction machine.

Under certain conditions the induction machine, taking 8 MVA at 85% power factor, will act as a motor to drive the compressor-expander to compress a certain gas.

At other times, the compressor-expander will act as a turbine to drive the electric machine as a generator supplying 6 MW at 82% power factor.

Also connected to the plant bus:

Induction motors taking 4 MVA at 86% pf

Incandescent lighting taking 2MW

Synchronous motor taking 2 MW at 90% pf

The plant will receive the necessary power from the utility at 13.8kV

REQUIREMENTS:

In order to formulate the conceptual design of the new substation, determine the plant load in MW, amperes and power factor that the utility must supply under the following conditions:

a) When the induction machine is driving the compressor-expander.

b) When the compressor-expander is driving the induction machine.

SOLUTION:

Keep track of P's and Q's – Solve for S, I and pf

System – Induction motor driving compressor expander

	S	P	Q	Pf
Induction Motors	4			0.86
Lighting		2		1.0
Synchronous Motors		2		0.9 leading
Compressor-Expander	8			0.85
Plant				

Induction motors:

$$S = 4MVA$$
$$pf = 0.86$$
$$P = 4MVA * 0.86 = 3.44MW$$
$$Q = \sqrt{4^2 - 3.44^2} = 2.041MVAR$$

Lighting

$$P = 2MW$$
$$pf = 1.0$$
$$S = \frac{2MW}{1.0} = 2.0MVA$$

$$Q = \sqrt{1^2 - 1^2} = 0MVAR$$

Synchronous Motors

$$P = 2MW$$
$$pf = 0.9(leading)$$
$$S = \frac{2MW}{0.9} = 2.222MVA$$
$$Q = \sqrt{2.222^2 - 2^2} = -0.9686MVAR$$

Compressor – Expander

$$pf = 0.85$$
$$S = 8MVA$$
$$P = \frac{8MVA}{0.85} = 6.8MW$$
$$Q = \sqrt{8^2 - 6.8^2} = 4.215MVAR$$

	S	P	Q	Pf
Induction Motors	4	3.44	2.041	0.86
Lighting	2	2	0	1.0
Synchronous Motors	2.222	2	-0.9686	0.9 leading
Compressor-Expander	8	6.8	4.214	0.85
Plant		14.24	5.286	

a)

$$P_{plant} = 14.24MW$$
$$S_{plant} = 14.24 + j5.286 = 15.189\measuredangle 20.37$$
$$pf = \cos 20.37^\circ = 0.9375$$

$$S = \sqrt{3}VI^* \Rightarrow$$
$$I^* = \frac{S}{\sqrt{3}V}$$
$$= \frac{15.189MVA}{\sqrt{3} * 13.8kV} = 635.5Amp$$

b) Same as above, but compressor-expander driving induction generator

Compressor – Expander

$$P = -6.0MW$$
$$pf = 0.82$$
$$S = \frac{-6.0}{0.82} = -7.317MVA$$
$$Q = \sqrt{(-7.317)^2 - (-6.0)^2} = 4.188MVAR$$

	S	P	Q	Pf
Induction Motors	4	3.44	2.041	0.86
Lighting	2	2	0	1.0
Synchronous Motors	2.222	2	-0.9686	0.9 leading
Compressor-Expander	-7.317	-6.0	4.188	0.85
Plant		1.44	5.26	

$$P_{plant} = 1.44MW$$
$$S_{plant} = 1.44 + j5.260 = 5.454\measuredangle 74.69^\circ$$
$$pf = \cos 74.69^\circ = 0.264$$

$$I^* = \frac{S}{\sqrt{3}V}$$
$$= \frac{5.454MVA}{\sqrt{3}*13.8kV} = 228.2Amp$$

9.5 Applications

Applications are an opportunity to demonstrate familiarity, comfort, and comprehension of the topics.